Exercise 32

Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g and state their domains.

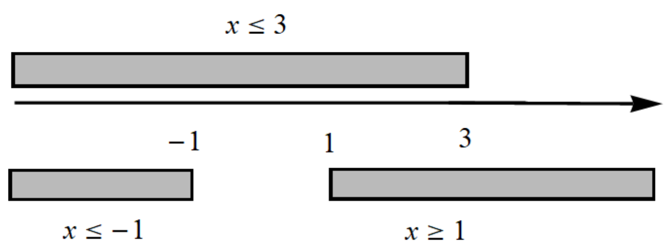
$$f(x) = \sqrt{3-x}, \quad g(x) = \sqrt{x^2-1}$$

Solution

$$\begin{aligned} f+g &= f(x)+g(x) \\ &= \sqrt{3-x} + \sqrt{x^2-1} \end{aligned}$$

The domain of a square root is found by requiring the expression under it be greater than or equal to zero. Since there are two, chain them together with "and."

$$\begin{aligned} 3-x \geq 0 &\quad \text{and} \quad x^2-1 \geq 0 \\ -x \geq -3 &\quad \text{and} \quad x^2 \geq 1 \\ x \leq 3 &\quad \text{and} \quad (x \leq -1 \quad \text{or} \quad x \geq 1) \end{aligned}$$



The domain of $f+g$ is where the top and bottom of the number line both have grayness.

$$\{x \mid x \leq -1 \ \cup \ 1 \leq x \leq 3\}$$

$$\begin{aligned} f-g &= f(x)-g(x) \\ &= \sqrt{3-x} - \sqrt{x^2-1} \end{aligned}$$

The domain of $f-g$ is the same as before.

$$\{x \mid x \leq -1 \ \cup \ 1 \leq x \leq 3\}$$

$$\begin{aligned} fg &= f(x)g(x) \\ &= \sqrt{3-x}\sqrt{x^2-1} \\ &= \sqrt{(3-x)(x^2-1)} \end{aligned}$$

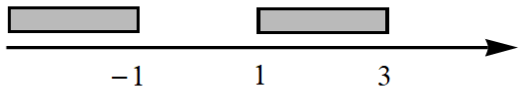
The domain of a square root is found by requiring the expression under it be greater than or equal to zero.

$$(3-x)(x^2-1) \geq 0$$

Factor the left side.

$$(3 - x)(x + 1)(x - 1) \geq 0$$

The critical points are $x = 3$, $x = -1$, and $x = 1$.



The grayness above the number line indicates where the above inequality is true. The domain of fg is then

$$\{x \mid x \leq -1 \,\cup\, 1 \leq x \leq 3\}.$$

$$\begin{aligned} \frac{f}{g} &= \frac{f(x)}{g(x)} \\ &= \frac{\sqrt{3 - x}}{\sqrt{x^2 - 1}} \end{aligned}$$

The domain of a square root is found by requiring the expression under it be greater than or equal to zero. Also, for a fraction, the denominator cannot be zero. Chain the expressions together with "and."

$$\begin{gathered} x^2 - 1 \geq 0 \quad \text{and} \quad \sqrt{x^2 - 1} \neq 0 \\ x^2 - 1 \geq 0 \quad \text{and} \quad x^2 - 1 \neq 0 \\ x^2 - 1 > 0 \\ x^2 > 1 \\ x < -1 \quad \text{or} \quad x > 1 \end{gathered}$$

The domain of f/g is then

$$\{x \mid x < -1 \,\cup\, x > 1\}.$$